AB Calculus Review Sheet #2

Rules Review the rules for differentiation: Sum, product, quotient, chain, and power.
Review the rules for differentiation of special functions: trig, exponential, general inverses, inverse trig, logs and natural logs, implicitly defined functions, and functions defined by integrals.
Review rules for definite and indefinite integrals.
Review rules for integration by parts.
Review writing the equation of a tangent line.
Review using a tangent line as a linear approximation of a function.
Create your own list of rules!!!!!!!

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1. If $y=(4x+1)(1-x)^3$ then $\frac{dy}{dx}=$

(A) $-12(1-x)^3$ (B) $(1-x)^2(1+8x)$ (C) $(1-x)^2(1-16x)$

(D) $3(1-x)^2(4x+1)$ (E) $(1-x)^2(16x+7)$

2. If $y=\frac{2-x}{3x+1}$ then $\frac{dy}{dx}=$

(A) $-\frac{7}{(3x+1)^2}$ (B) $\frac{6x-5}{(3x+1)^2}$ (C) $-\frac{9}{(3x+1)^2}$ (D) $\frac{7}{(3x+1)^2}$ (E) $\frac{7-6x}{(3x+1)^2}$

3. If $y=\sqrt{3-2x}$ then $\frac{dy}{dx}=$

(A) $\frac{1}{2\sqrt{3-2x}}$ (B) $-\frac{1}{\sqrt{3-2x}}$ (C) $-\frac{(3-2x)^{3/2}}{3}$ (D) $-\frac{1}{3-2x}$ (E) $\frac{2}{3}(3-2x)^{3/2}$

4. The normal to the curve represented by the equation $y=x^2+6x+4$ at the point $(-2,-4)$ also intersects the curve at $x=$

(A) -6 (B) $-\frac{9}{2}$ (C) $-\frac{7}{2}$ (D) -3 (E) $-\frac{1}{2}$

5. Find y' given $y=\ln\frac{e^x}{e^x-1}$.

(A) $x-\frac{e^x}{e^x-1}$ (B) $\frac{1}{e^x-1}$ (C) $-\frac{1}{e^x-1}$ (D) 0 (E) $\frac{e^x-2}{e^x-1}$

In questions 6 through 10, differentiable function f and g have the values shown the in table.

x	f	f'	g	g'
0	2	1	5	–4
1	3	2	3	–3
2	5	3	1	–2
3	10	4	0	–1

6. If $B = f \cdot g$, then $B'(2) =$

(A) –20 (B) –7 (C) –6 (D) –1 (E) 13

7. If $H(x) = \sqrt{f(x)}$, then $H'(3) =$

(A) $\frac{1}{4}$ (B) $\frac{1}{2\sqrt{10}}$ (C) 2 (D) $\frac{2}{\sqrt{10}}$ (E) $4\sqrt{10}$

8. If $D = \frac{1}{g}$, then $D'(1) =$

(A) $-\frac{1}{2}$ (B) $-\frac{1}{3}$ (C) $-\frac{1}{9}$ (D) $\frac{1}{9}$ (E) $\frac{1}{3}$

9. If $M(x) = f(g(x))$, then $M'(1) =$

(A) -12 (B) -6 (C) 4 (D) 6 (E) 12

10. If $S(x) = f^{-1}(x)$, then $S'(3) =$

(A) -2 (B) $-\frac{1}{25}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) 2

11. A differentiable function f has values shown. Estimate $f'(1.5)$.

x	1.0	1.2	1.4	1.6
$f(x)$	8	10	14	22

(A) 8 (B) 12 (C) 18 (D) 40 (E) 80

12. Find $\frac{dy}{dx}$ given $y = \ln(\sec x + \tan x)$.

(A) $\sec x$ (B) $\frac{1}{\sec x}$ (C) $\tan x + \frac{\sec^2 x}{\tan x}$

(D) $\frac{1}{\sec x + \tan x}$ (E) $-\frac{1}{\sec x + \tan x}$

13. Find $\frac{dy}{dx}$ given $y = \sin^{-1} x - \sqrt{1-x^2}$

(A) $\frac{1}{2\sqrt{1-x^2}}$ (B) $\frac{2}{\sqrt{1-x^2}}$ (C) $\frac{1+x}{\sqrt{1-x^2}}$

(D) $\frac{x^2}{\sqrt{1-x^2}}$ (E) $\frac{1+x}{\sqrt{1+x}}$

14. If $x^3 - xy + y^3 = 1$, then $\frac{dy}{dx} =$

(A) $\frac{3x^2}{x-3y^2}$ (B) $\frac{3x^2-1}{1-3y^2}$ (C) $\frac{y-3x^2}{3y^2-x}$

(D) $\frac{3x^2+3y^2-y}{x}$ (E) $\frac{3x^2+3y^2}{x}$

15. $\int \frac{x\,dx}{1+4x^2} =$

(A) $\frac{1}{8}\ln(1+4x^2)+C$ (B) $\frac{1}{8(1+4x^2)^2}+C$ (C) $\frac{1}{4}\sqrt{1+4x^2}+C$

(D) $\frac{1}{2}\ln(1+4x^2)+C$ (E) $\frac{1}{2}\tan^{-1}(2x)+C$

16. $\int \frac{dx}{1+4x^2} =$

(A) $\tan^{-1}(2x)+C$ (B) $\frac{1}{8}\ln(1+4x^2)+C$ (C) $\frac{1}{8(1+4x^2)^2}+C$

(D) $\frac{1}{2}\tan^{-1}(2x)+C$ (E) $\frac{1}{8x}\ln(1+4x^2)+C$

17. $\int \frac{x}{\left(1+4x^2\right)^2}\,dx =$

(A) $\frac{1}{8}\ln(1+4x^2)^2+C$ (B) $\frac{1}{4}\sqrt{1+4x^2}+C$ (C) $-\frac{1}{8(1+4x^2)}+C$

(D) $-\frac{1}{3\left(1+4x^2\right)^3}+C$ (E) $-\frac{1}{\left(1+4x^2\right)}+C$

18. $\int \frac{x\,dx}{\sqrt{1+4x^2}} =$

(A) $\frac{1}{8}\sqrt{1+4x^2}+C$ (B) $\frac{\sqrt{1+4x^2}}{4}+C$ (C) $\frac{1}{2}\sin^{-1}(2x)+C$

(D) $\frac{1}{2}\tan^{-1}(2x)+C$ (E) $\frac{1}{2}\ln\sqrt{1+4x^2}+C$

19. $\int(\sin x-3\cot x\sin x)\,dx =$

(A) $-\cos x-3\sin x+C$ (B) $\cos x+3\sin x+C$ (C) $\cos x+3\csc x+C$

(D) $-\cos x+3\csc x+C$ (E) none of these

20. $\int \frac{x}{\cos^2(3x^2)}\,dx =$

(A) $\frac{1}{3}\arccos(3x^2)+C$ (B) $\frac{1}{6}\tan(3x^2)+C$ (C) $-\frac{1}{6}\tan(3x^2)+C$

(D) $\frac{1}{3}\sec(3x^2)+C$ (E) none of these

21. $\int \tan(5x)\,dx =$

(A) $5\sec^2 5x+C$ (B) $-\frac{1}{5}\ln|\cos 5x|+C$ (C) $-\frac{1}{5}\ln|\sec 5x|+C$

(D) $-5\ln|\cos 5x|+C$ (E) $\frac{1}{5}\ln|\tan 5x|+C$

22. If $y = 2^{5x^2}$, then $\frac{dy}{dx} =$

(A) $10x \cdot 2^{5x^2}$ (B) $e^{5x^2 \ln 2}$ (C) $10x \cdot \ln 2 \cdot 2^{5x^2}$ (D) 2^{5x^2} (E) $2^{5x^2} \cdot \ln 2$

23. If $y = \log_3 4x$, then $\frac{dy}{dx} =$

(A) $\frac{1}{x}$ (B) $\frac{1}{12x}$ (C) $\frac{1}{4x \ln 3}$ (D) $\frac{1}{x \ln 3}$ (E) $\frac{1}{4x}$

Free Response Questions:

1. Given the function f defined by $f(x) = x^3 - x^2 - 4x + 4$.

(a) Find the zeros of f.

(b) Write an equation of the line tangent to the graph of f at $x = -1$.

(c) The point (a, b) is on the graph of f and the tangent to the graph at (a, b) passes through the point $(0, -8)$ which is not on the graph of f. Find the values of a and b.

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2. Given the curve $x + xy + 2y^2 = 6$.

(a) Find an expression for the slope of the curve at any point (x, y) on the curve.

(b) Write an equation for the line tangent to the curve at the point $(2, 1)$.

(c) Find the coordinates of all other points on this curve with slope equal to the slope at $(2, 1)$.

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3. Let f be the real-valued function defined by $f(x) = \sqrt{1 + 6x}$.

(a) Give the domain and range of f.

(b) Determine the slope of the line tangent to the graph of f at $x = 4$.

(c) Determine the y-intercept of the line tangent to the graph of f at $x = 4$.

(d) Give the coordinates of the point on the graph of f where the tangent line is parallel to $y = x + 12$.

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4. Let f be the function given by $f(x) = \dfrac{|x| - 2}{x - 2}$.

(a) Find all the zeros of f.

(b) Find $f'(1)$.

(c) Find $f'(-1)$.

(d) Find the range of f.

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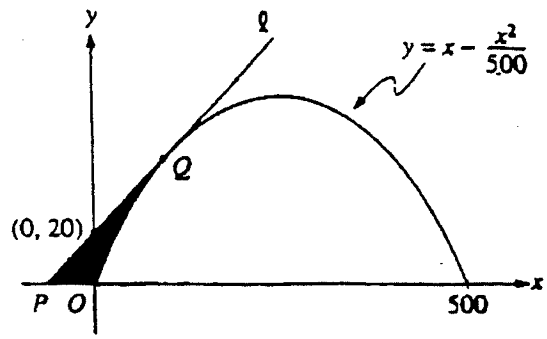
5. Line l is tangent to the graph of $y = x - \dfrac{x^2}{500}$ at the point Q, as shown in the figure below.



(a) Find the x-coordinate of point Q.

(b) Write an equation for line l.

(c) Suppose the graph of $y = x - \dfrac{x^2}{500}$ shown in the figure, where x and y are measured in feet, represents a hill. There is a 50-foot tree growing vertically at the top of the hill. Does a spotlight at point P directed along line l shine on any part of the tree? Show the work that leads to your conclusion.

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6. Consider the curve defined by the equation $y + \cos y = x + 1$ for $0 \le y \le 2\pi$.

(a) Find $\dfrac{dy}{dx}$ in terms of y.

(b) Write an equation for each vertical tangent to the curve.

(c) Find $\dfrac{d^2y}{dx^2}$ in terms of y.

Answers for AB Calculus AP Review

Review Sheet #2

1. C	7. D	13. C	19. A
2. A	8. E	14. C	20. B
3. B	9. A	15. A	21. B
4. B	10. D	16. D	22. C
5. C	11. D	17. C	23. D
6. B	12. A	18. B	

1. 1978 AB1
2. 1973 AB3
3. 1976 AB1
4. 1991 AB4
5. 1996 AB6
6. 1992 AB4
7. 1979 AB1
8. 1995 AB3

Given the function f defined by $f(x) = x^3 - x^2 - 4x + 4$.

(a) Find the zeros of f.

(b) Write an equation of the line tangent to the graph of f at $x = -1$.

(c) The point (a,b) is on the graph of f and the line tangent to the graph at (a,b) passes through the point (0,-8) which is not on the graph of f. Find the value of a and b.

(a) $x^3 - x^2 - 4x + 4 = 0$

$x^2(x-1) - 4(x-1) = 0$

$(x-1)(x^2-4) = 0$

$x = 1, \pm 2$

(b) $f'(x) = 3x^2 - 2x - 4$

$f'(-1) = 1$

$f(-1) = 6$

$y - 6 = 1(x+1)$

$x - y + 7 = 0$

(c) line contains (a,b), (0,-8) & has slope $f'(a) \Rightarrow$

$$\frac{b+8}{a} = 3a^2 - 2a - 4$$

$f(a) = b \Rightarrow \quad a^3 - a^2 - 4a + 4 = b$

$3a^3 - 2a^2 - 4a - 8 = b$

$2a^3 - a^2 - 12 = 0$

$(a-2)(2a^2 + 3a + 6) = 0$

$a = 2$

2	2	-1	0	-12
		4	6	12
	2	3	6	0

$f(2) = 8 - 4 - 8 + 4 = 0$

$\therefore$ a = 2 and b = 0

Given the curve $x + xy + 2y^2 = 6$.

(a) Find an expression for the slope of the curve at any point (x,y) on the curve.
(b) Write an equation for the line tangent to the curve at the point $(2,1)$.
(c) Find the coordinates of all other points on this curve with slope equal to the slope at $(2,1)$.

(a) $x + xy + 2y^2 = 6$

$$1 + x\frac{dy}{dx} + y + 4y\frac{dy}{dx} = 0$$

$$(x+4y)\frac{dy}{dx} = -(1+y)$$

$$\frac{dy}{dx} = -\frac{1+y}{x+4y}$$

(b) $$\left.\frac{dy}{dx}\right|_{(2,1)} = -\frac{1+1}{2+4} = -\frac{1}{3}$$

$$\therefore\ y-1 = -\frac{1}{3}(x-2)$$

$$x + 3y = 5$$

(c) $$-\frac{1+y}{x+4y} = -\frac{1}{3}$$

$$3 + 3y = x + 4y$$

$$y = 3 - x$$

$$\therefore\ x + x(3-x) + 2(3-x)^2 = 6$$

$$x^2 - 8x + 12 = 0$$

$$(x-6)(x-2) = 6$$

$$x = 6, 2$$

$x = 6 \Rightarrow y = -3$ Other point is $(6,-3)$

Let f be a real-valued function defined by $f(x) = \sqrt{1 + 6x}$ ..

(a) Give the domain and range of f.
(b) Determine the slope of the line tangent to the graph of f at $x = 4$.
(c) Determine the y-intercept of the line tangent to the graph of f at $x = 4$.
(d) Give the coordinates of the point on the graph of f where the tangent line is parallel to $y = x + 12$.

(a) Domain of $f = \{x : x \geq -\frac{1}{6}\}$

Range of $f = \{y : y \geq 0\}$

(b) $f'(x) = \dfrac{3}{\sqrt{1+6x}}$

$f'(4) = \dfrac{3}{5}$

(c) $f(4) = 5$

$y - 5 = \frac{3}{5}(x-4)$

$y = \frac{3}{5}x + \frac{13}{5}$

y intercept is $\frac{13}{5}$

(d) tangent line $\parallel$ to $y = x + 12 \Rightarrow f'(x) = 1$

$\dfrac{3}{\sqrt{1+6x}} = 1$

$9 = 1 + 6x$

$x = 4/3$

$f(4/3) = 3$

$(4/3, 3)$

AB 4

1991

1991 AB4

4. Let f be the function given by $f(x) = \dfrac{|x| - 2}{x - 2}$.

(a) Find all the zeros of f.

(b) Find $f'(1)$.

(c) Find $f'(-1)$.

(d) Find the range of f.

(a) $f(x) = 0 \iff |x| = 2,\ x \neq 2$

$x = -2$

1: $x = -2$

(b) For $x \geq 0$, $x \neq 2$

$$f(x) = \frac{x-2}{x-2} = 1$$

$\therefore f'(1) = 0$

1: $f'(1) = 0$

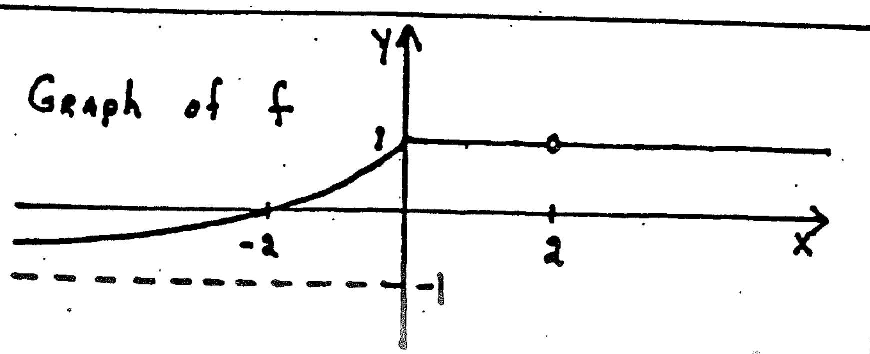
(c) For $x < 0$, $f(x) = \dfrac{-x-2}{x-2}$

$$f'(x) = \frac{(x-2)(-1) - (-x-2)(1)}{(x-2)^2} = \frac{4}{(x-2)^2}$$

$\therefore f'(-1) = \dfrac{4}{9}$

4: $\begin{cases} f'(x) = \dfrac{4}{(x-2)^2} \\ f'(-1) = \dfrac{4}{9} \\ \text{-2: mishandled } |x| \text{ or its derivative for } x<0 \\ \text{-2: quotient rule error or no quotient rule} \\ \text{-1: incorrect or no computation of } f'(-1) \\ \text{-1: each other error} \end{cases}$

(d) $-1 < y \leq 1$



1: lub $y = 1$

1: glb $y = -1$

1: correct inequalities at -1 and 1

1/3 for $\{-1, 1\}$

0/3 for other finite sets

1996 AB 6

5

Solution

Scoring Scale

Points

(a) let Q be $\left(a, a - \frac{a^2}{500}\right)$.

$$\frac{dy}{dx} = 1 - \frac{x}{250}$$

setting slopes equal:

$$1 - \frac{a}{250} = \frac{\left(a - \frac{a^2}{500}\right) - 20}{a}$$

$a = 100$

or

$$\frac{dy}{dx} = 1 - \frac{x}{250}$$

equation for ℓ : $y = \left(1 - \frac{a}{250}\right)x + 20$

setting y-values equal:

$$\left(1 - \frac{a}{250}\right)a + 20 = a - \frac{a^2}{500}$$

$a = 100$

4:
- 1 : slope of tangent line from parabola
- 1 : uses the condition that $\left(a, a - \frac{a^2}{500}\right)$ is on line ℓ
- 1 : uses the condition that slopes are equal at Q
- 1 : answer
  0/1 if student is solving an irrelevant equation

(b) $y = \frac{3}{5}x + 20$

2:
- 1 : slope
  0/1 if $m \leq 0$
- 1 : equation

(c) height of hill at $x = 250$:

$$y = 250 - \frac{250^2}{500} = 125 \text{ feet}$$

height of line at $x = 250$:

$$y = \frac{3}{5}(250) + 20 = 170 \text{ feet}$$

Yes, the spotlight hits the tree since the height of the line is less than the height of the hill + tree which is 175 feet.

3:
- 1 : height of hill
- 1 : height of line
  0/1 if height < 0
- 1 : answer with analysis

AB-4
BC-1
1992

Consider the curve defined by the equation $y + \cos y = x + 1$ for $0 \le y \le 2\pi$.

(a) Find $\frac{dy}{dx}$ in terms of y.

(b) Write an equation for each vertical tangent to the curve.

(c) Find $\frac{d^2y}{dx^2}$ in terms of y.

(a) $\frac{dy}{dx} - \sin y \frac{dy}{dx} = 1$

$\frac{dy}{dx}(1 - \sin y) = 1$

$\frac{dy}{dx} = \frac{1}{1 - \sin y}$

3 { 2: Implicit Differentiation
<-1> if not with respect to x (may recoup from part (a) only
<-2> for chain rule error or incorrect differentiation of RH
<-1> $dy/dx + \sin y \, dy/dx = 1$
1: Solves for dy/dx
<-1> no factoring required }

(b) $\frac{dy}{dx}$ undefined when $\sin y = 1$

$y = \pi/2$

$\pi/2 + 0 = x + 1$

$x = \pi/2 - 1$

3 { Finds y where dy/dx does not exist
2 { 1: Establishes equation that determines where student
dy/dx does not exist
1: Solves that equation for y [must involve trig functi }
1: Uses that y solution to give equation of vertical l
<-1> Student not dealing with undefined dy/dx

Note:
1/3 if student's dy/dx always exists and student says there are no vertical tangents
max 2/3 if $dy/dx = \frac{1}{1 - \sin x}$ }

(c) $\frac{d^2y}{dx^2} = \frac{d\left(\frac{1}{1 - \sin y}\right)}{dx}$

$= \frac{-\left(-\cos y \, \frac{dy}{dx}\right)}{(1 - \sin y)^2}$

$= \frac{\cos y \, \frac{1}{1 - \sin y}}{(1 - \sin y)^2}$

$= \frac{\cos y}{(1 - \sin y)^3}$

3 { 2: Implicit Differentiation
<-2> if dy/dx is not a quotient or product in y
<-2> Any calculus error
1: Substitutes for student's dy/dx after differentiation and solves for d^2y/dx^2 }

4R